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WATCH Water and Global Change

Quantification of Europe-wide streamflow complexity

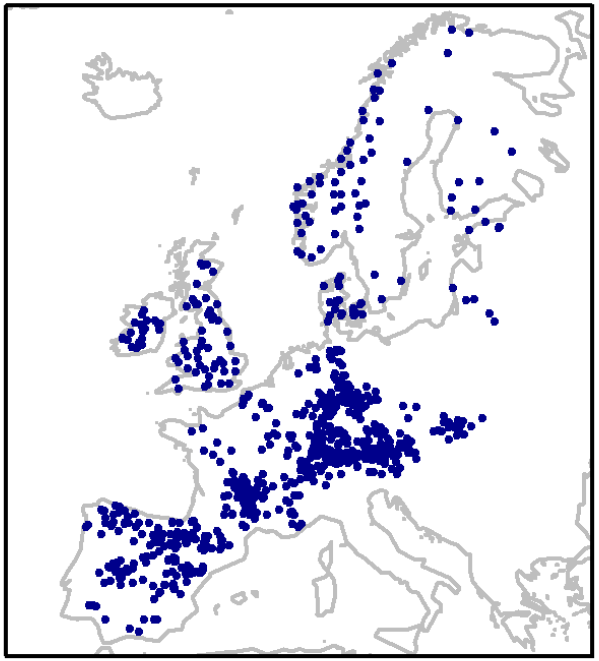
Lukas Gudmundsson

H. Lange , L.M. Tallaksen and K. Stahl

- **Linear variance decay – dimension density**
- **Technical issues**
- **Some examples**
  - Seasonality
  - links to atmospheric circulation patterns?

The Key Questions

- Multivariate Time Series
  - European Streamflow
- Spells of synchronous dynamics
  - When are things happening at the same time?
  - When are things different
- Why...?
  - Identification of processes leading to increased synchronisation



European Streamflow Series
- Daily observations
- 627 Time series
- 1962 – 2005

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Dimension of Multivariate Records

- Principal Component Analysis

$$\boldsymbol{S} = \boldsymbol{X}^T \boldsymbol{X} \qquad \boldsymbol{S} = \boldsymbol{U}^T \boldsymbol{\Sigma} \boldsymbol{U} \qquad \boldsymbol{\Sigma} = diag(\sigma_1^2, \ldots, \sigma_N^2)$$

- Number of relevant dimensions. f: variance threshold

$$D_{KLD}(f) = \min_p \{ [\sum_{i=1}^{p} \sigma_i^2 / \sum_{i=1}^{N} \sigma_i^2] \geq f \}$$

- Normalisation to sample size: **Dimension density**

$$\delta_{KLD} = D_{KLD} / N$$

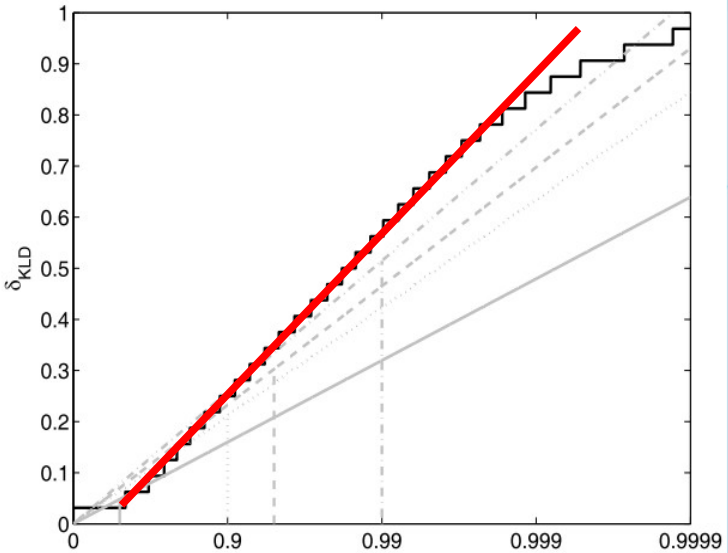
Linear Variance Decay Dimension Density

- Issues with δ_{LKD}:
  - Discrete: limited set of possible values

- Scaling of δ_{LKD} with f:
  - Scaling of remaining Variances (Φ)
  - Approximation: Exponential decay

$$\delta_{KLD}(\Phi) = -\delta_{LVD}(f)\log(1-\Phi)$$

  - Estimation: weighted least squares.



Donner, R. & Witt, A (2006)

Donner, R. & Witt, A. Nonlinear Processes In Geophysics, *2006*, 13, 485-497
Donner, R. et al. Nonlinear Time Series Analysis in the Geosciences, *2008*, 125-154

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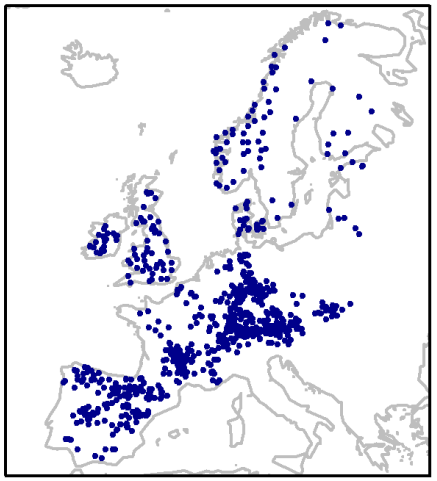
Some Technical Issues



More variables than observations:

$$\boldsymbol{S} = \boldsymbol{X}^T \boldsymbol{X}$$

Is ill conditioned.

Solution: Shrinkage $\boldsymbol{S}$ towards identity $\mathbf{1}$

$$\boldsymbol{S}^* = \kappa \mathbf{1} + (1-\kappa)\boldsymbol{S}$$

gives well conditioned estimates.

κ : found *analytically* to minimize quadratic loss using the Ledoit and Wolf theorem.

Ledoit, O. & Wolf, M. Journal of Empirical Finance, **2003**, 10, 603 - 621
Schäfer, J. & Strimmer, K.
Statistical Applications in Genetics and Molecular Biology, *2005*, 4, Article 32

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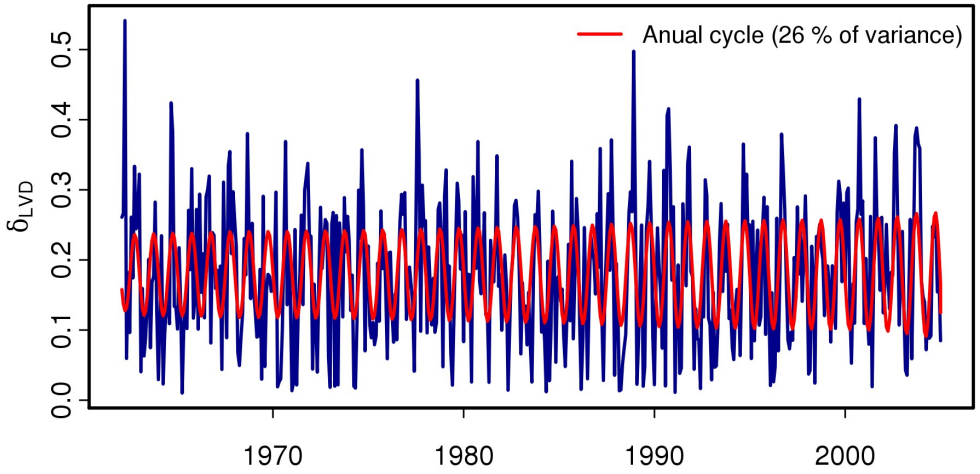
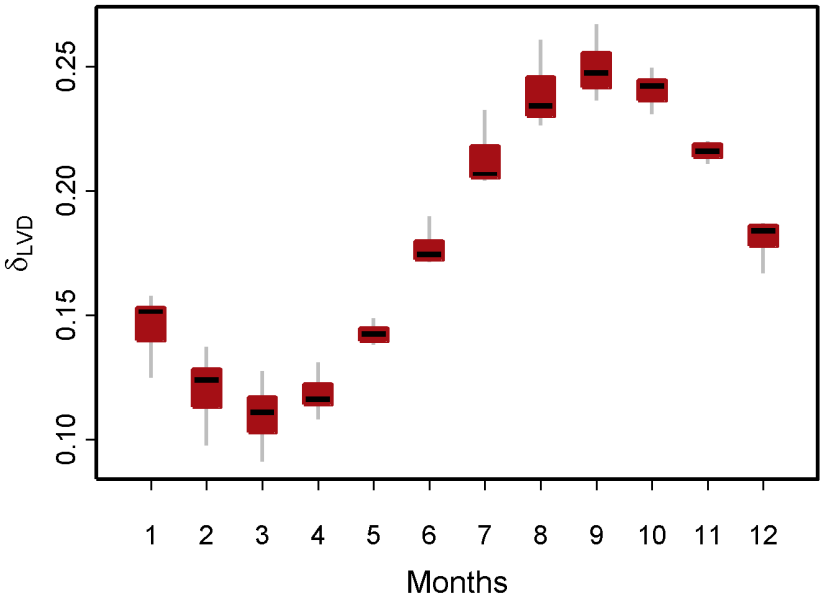
Monthly complexities



δ_{LVD}: monthly windows

Pronounced seasonality (26% of variance)

- **Minimum: Spring**
  - Strong *synchronisation* due to snow melt
- **Maximum: Late summer**
  - *Heterogeneous* weather patterns (Frontal and Convective systems)



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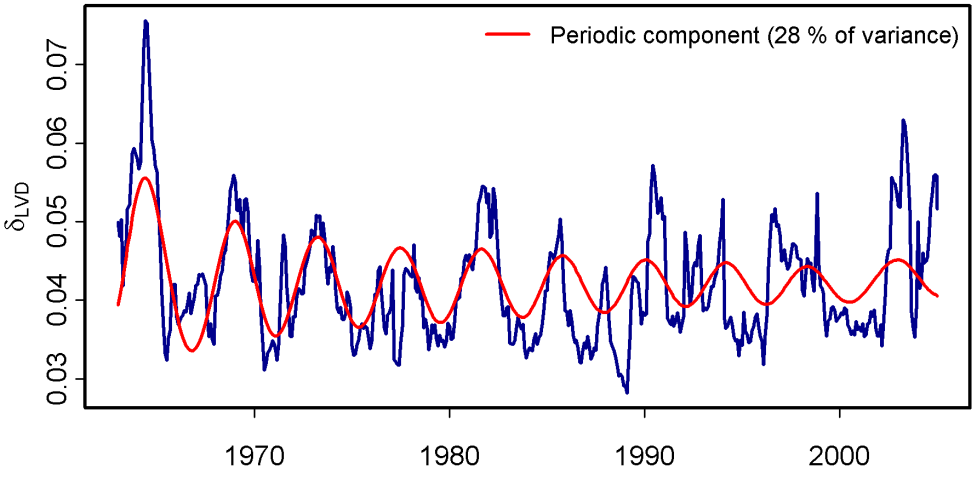
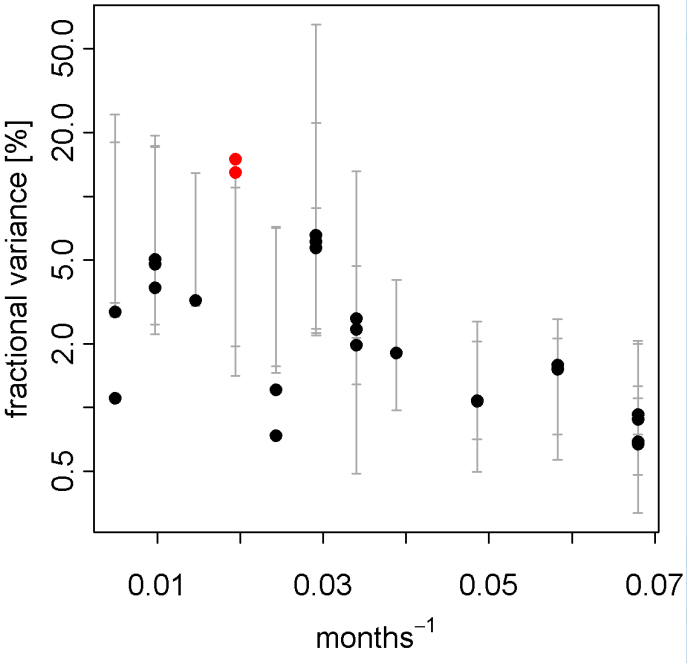
Moving annual complexities



δ_{LVD}: 365 – day window

- **MCSSA**
  - Time series decomposition
  - hypothesis based signal identification (AR1)
- **Long term component**
  - 28% of variance
  - Spectral power on **3** to **6** year band

Montecarlo Singular System Analysis (MCSSA)



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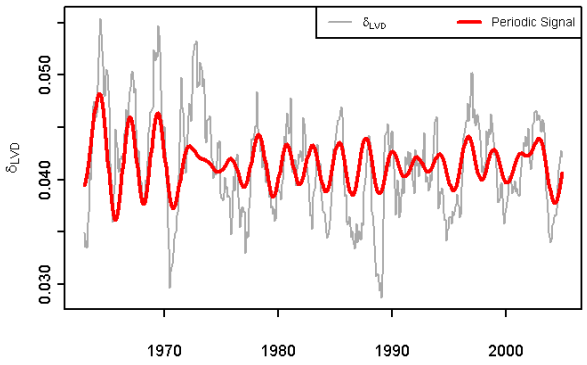
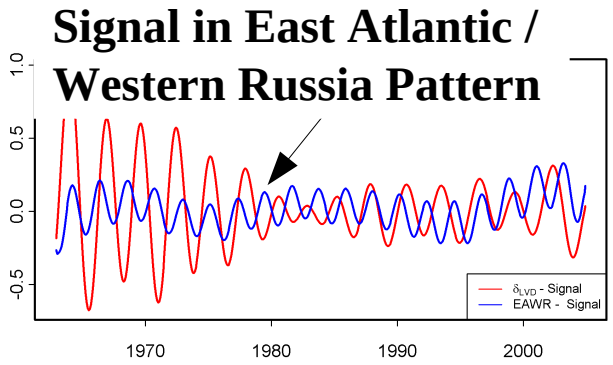
Sensitivity to the domain

We ***promised***: Relation to Teleconnections





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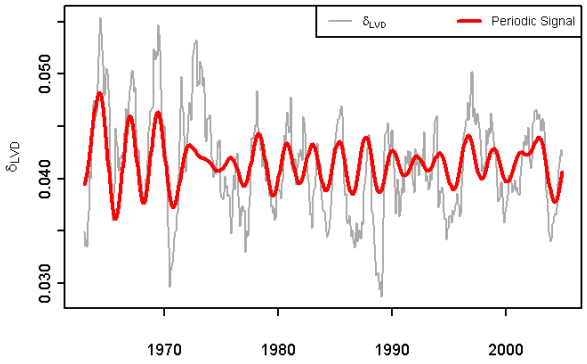
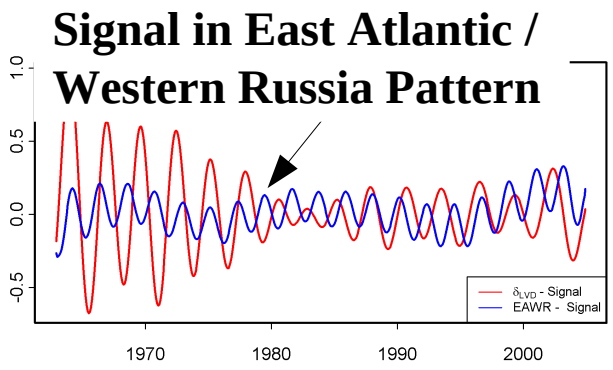
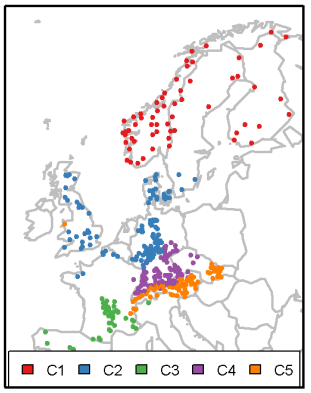
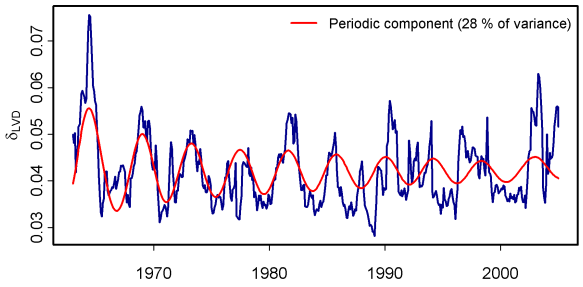
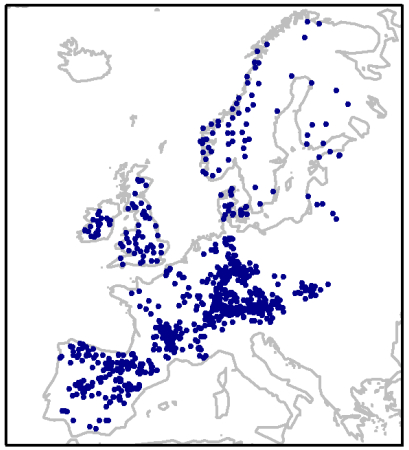






We **changed** the domain...





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Sensitivity to the domain

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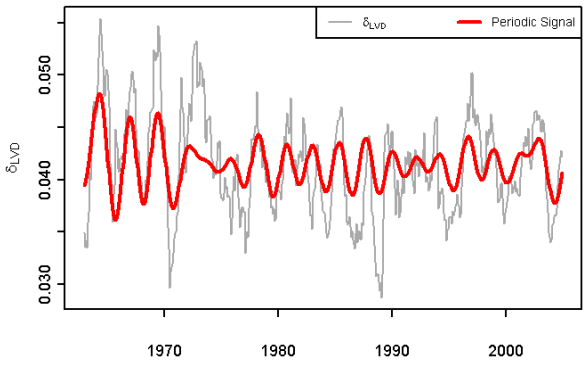

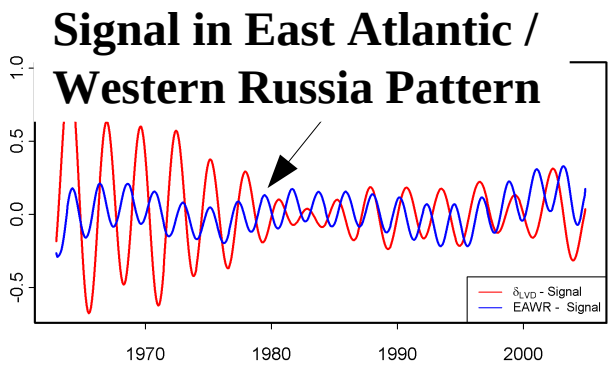

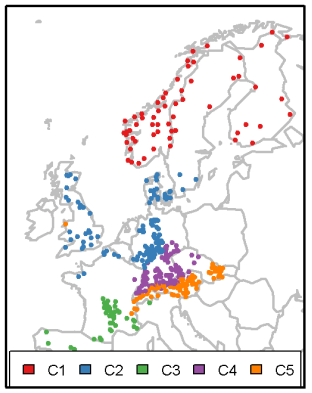

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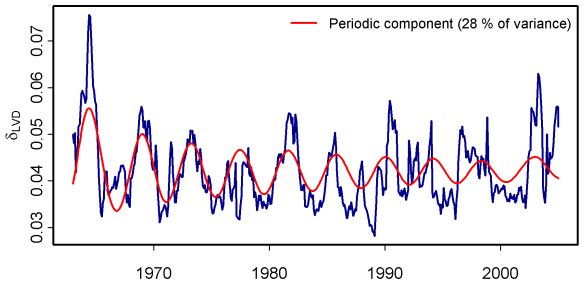

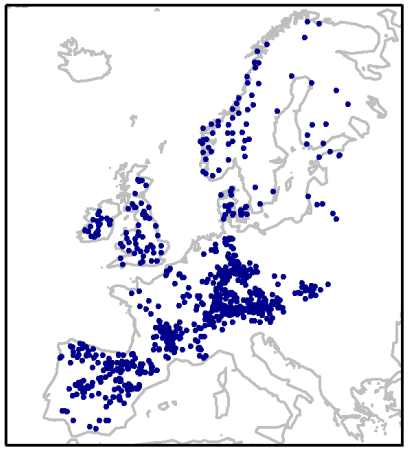

More on:

- *Regional* synchronisation
- *Extremes* vs. Means
- **HS2.5**
- Thursday, 23 Apr 2009
- 14:30–14:45

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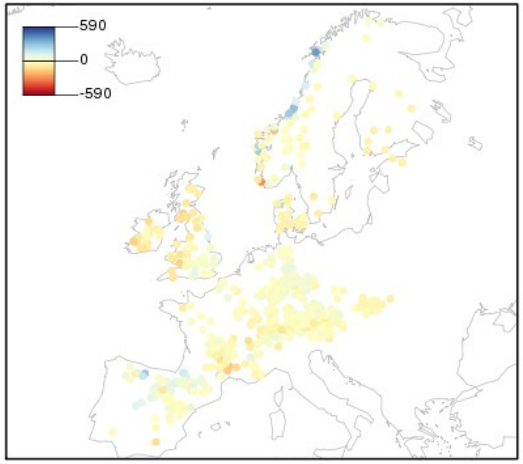
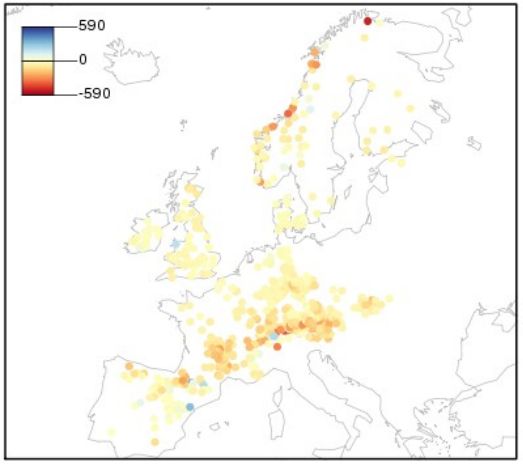
Dry conditions – low complexity?

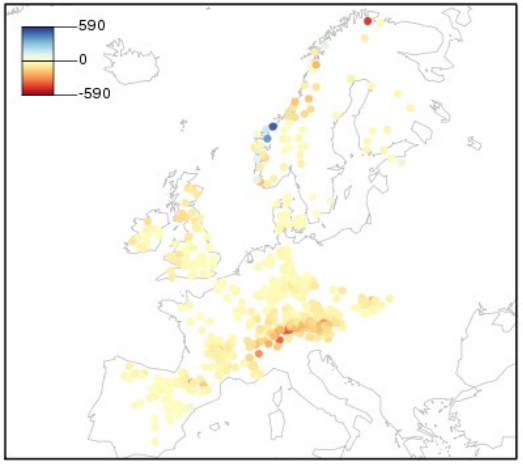
2003 Q1



2003 Q2



2003 Q3



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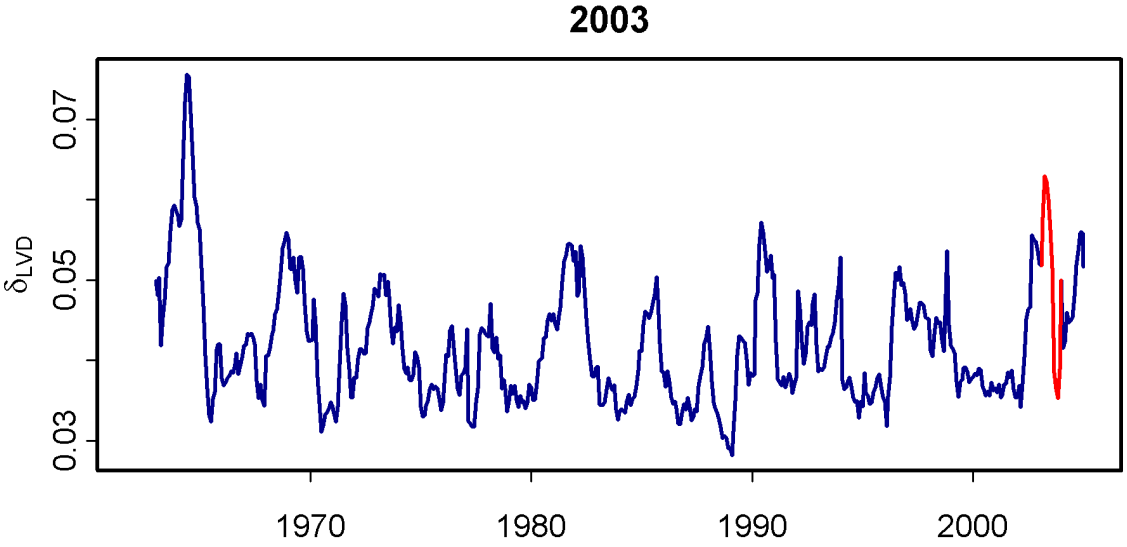
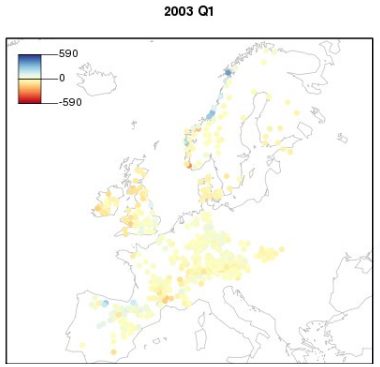
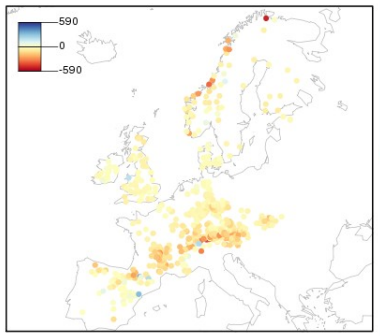
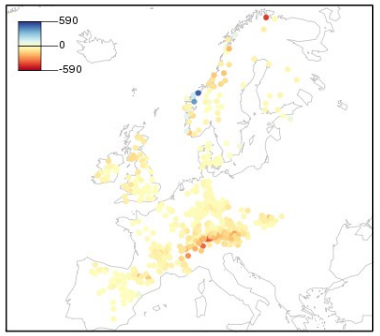
Drv conditions – low complexity?









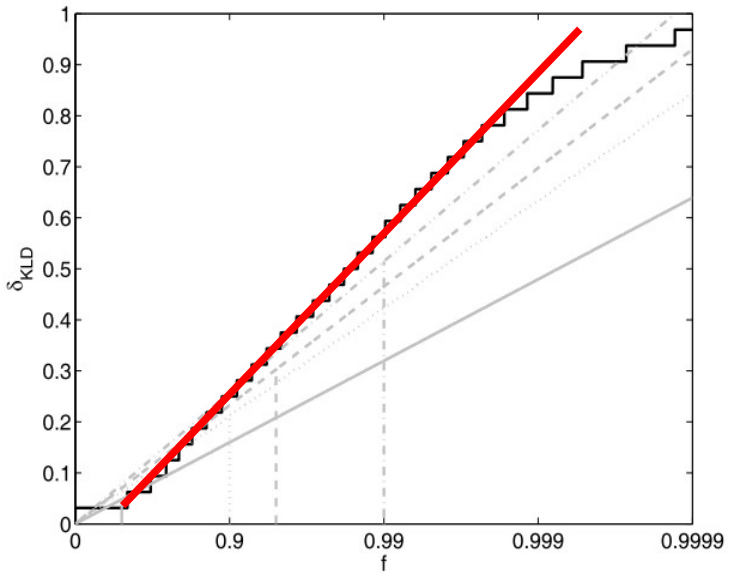
- **Onset of a drought: Drop in δ_{LVD}**
- **Dry and warm years:**
  - More homogeneous streamflow within the domain

More on droughts and climate:

- **Anne Fleig**
- **HS5.15**
- Fri, 24 Apr, 11:30–11:45

Summing up...

- δ_{LVD}: A robust estimator of the dimension density



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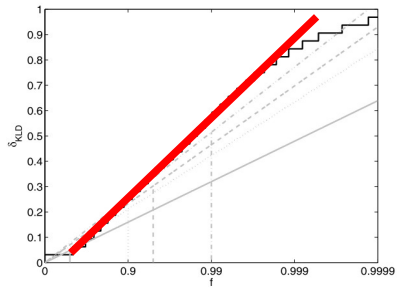
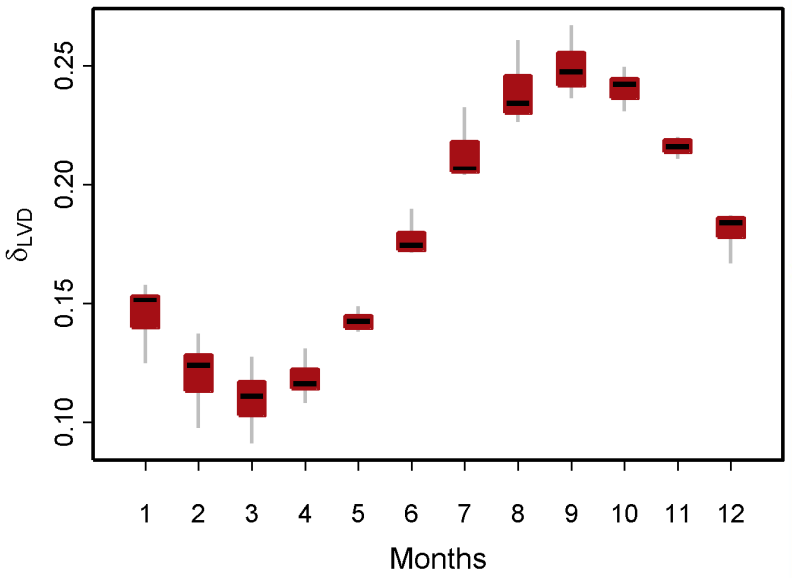
Summing up...

- δ_{LVD}:

  A robust estimator of the dimension density

  … Reveals seasonality in streamflow synchronisation





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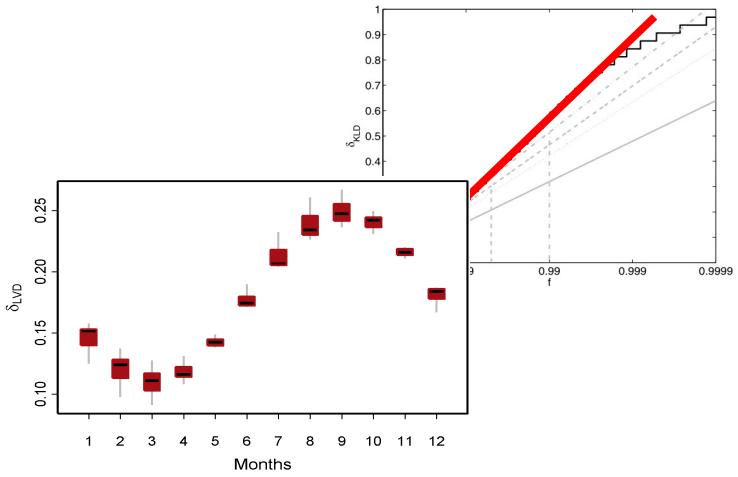
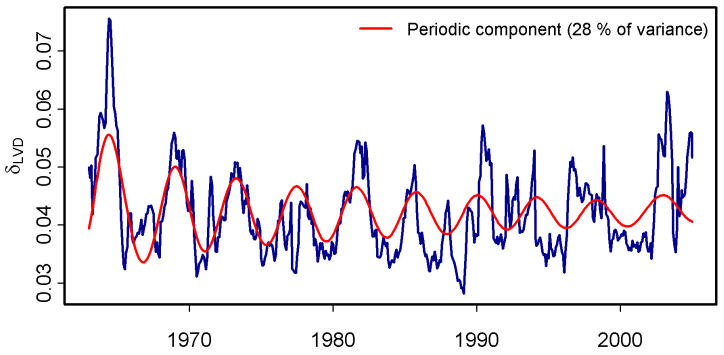
  A robust estimator of the dimension density



  … Reveals seasonality in streamflow synchronisation

  … Gives insights to long term dynamics



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Summing up...

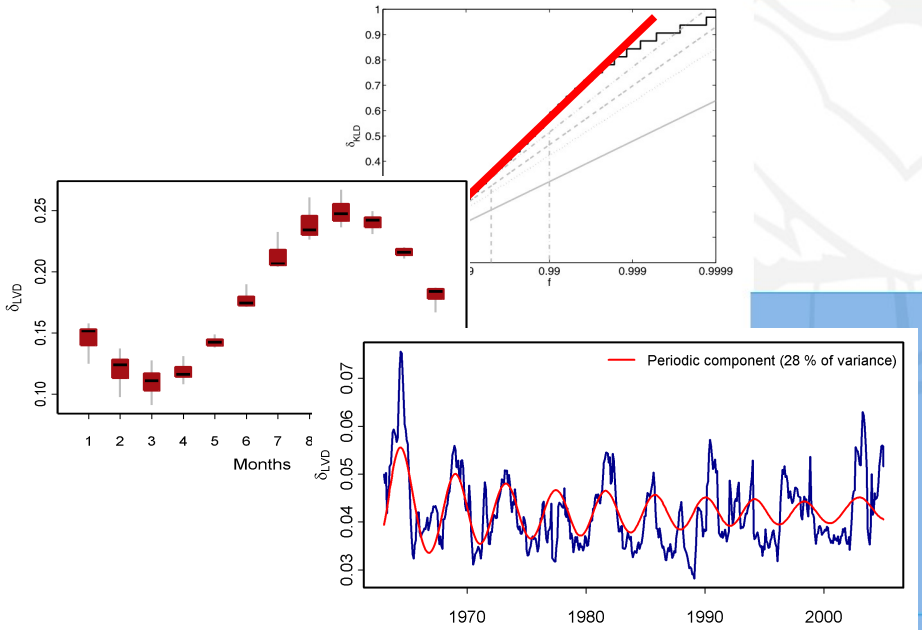
- δ_{LVD}:

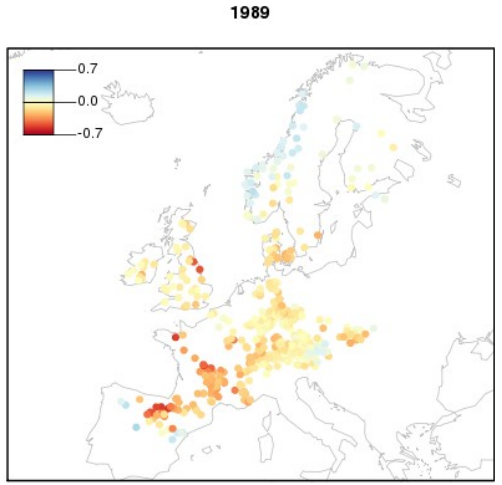
  A robust estimator of the dimension density

  … Reveals seasonality in streamflow synchronisation

  … Gives insights to long term dynamics

  … Useful to identify dry spells?





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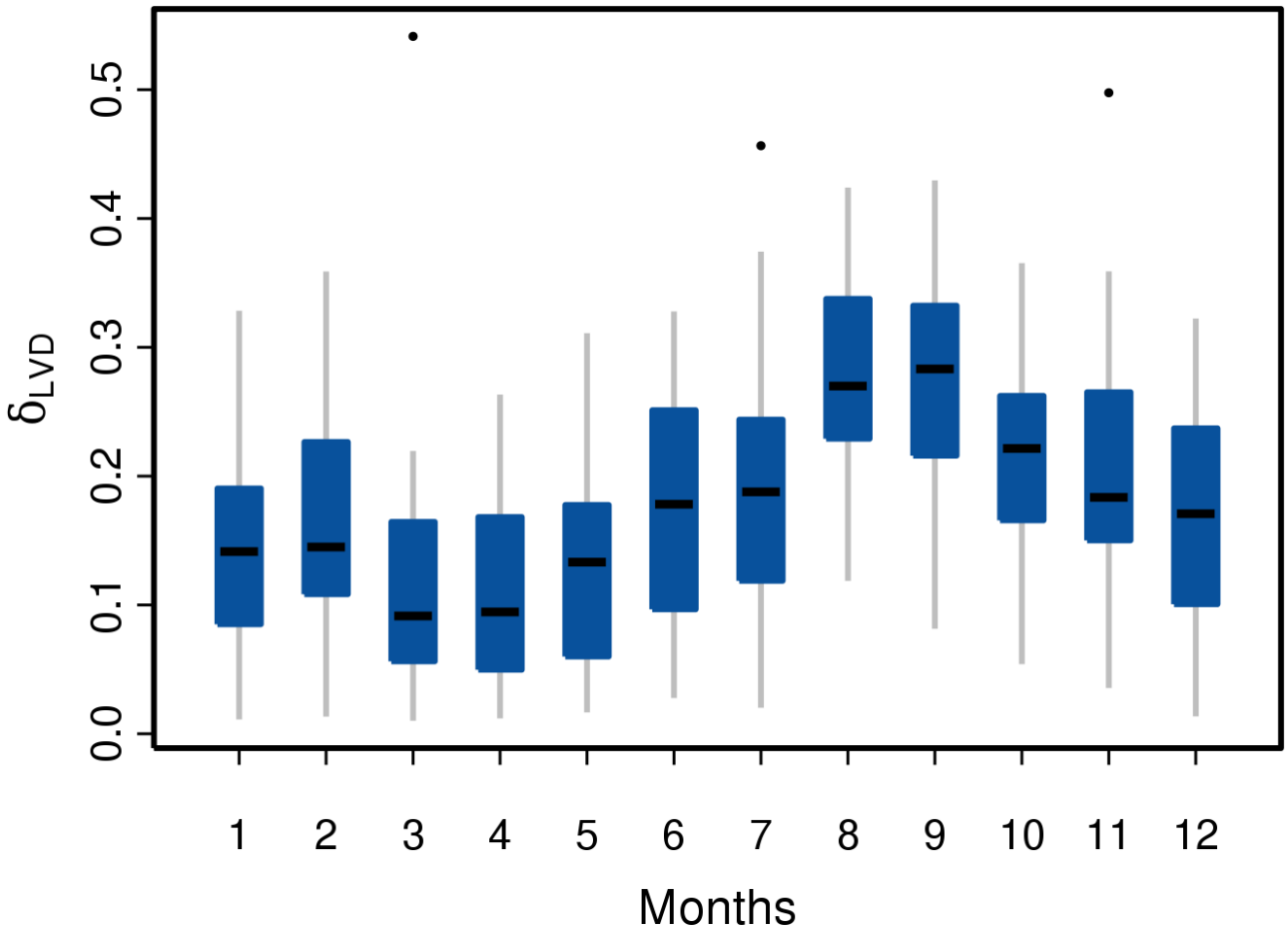
δ_{LVD}
0.0
0.1
0.2
0.3
0.4
0.5
1
2
3
4
5
6
7
8
9
10
11
12
Months

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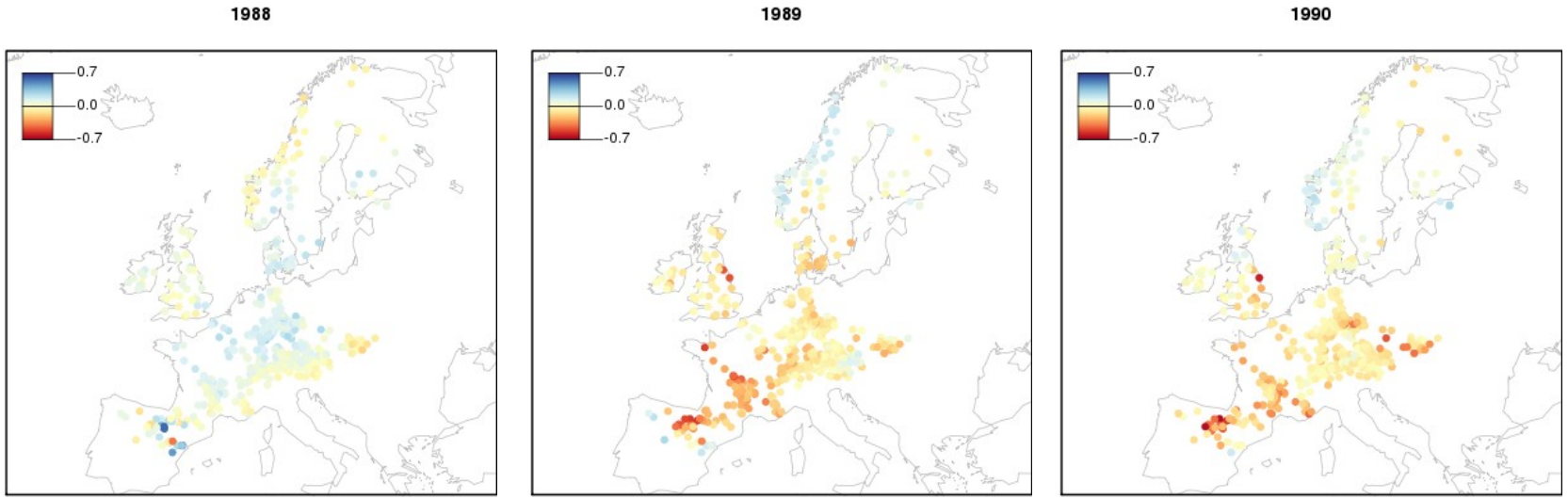
Dry conditions – low complexity?



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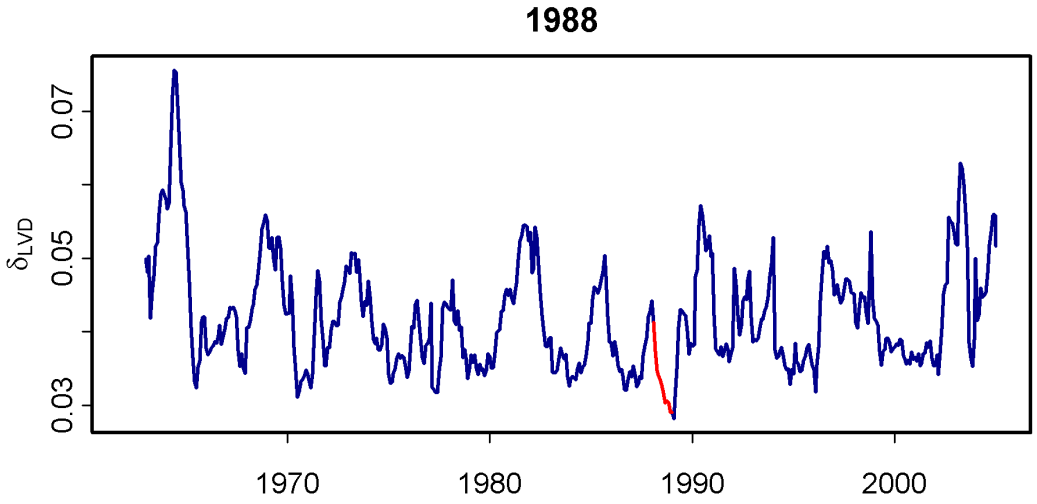

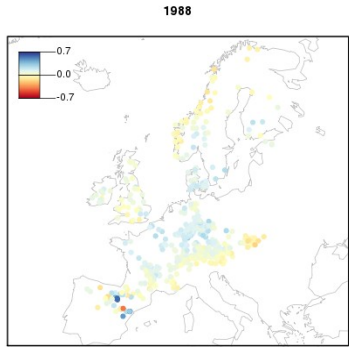

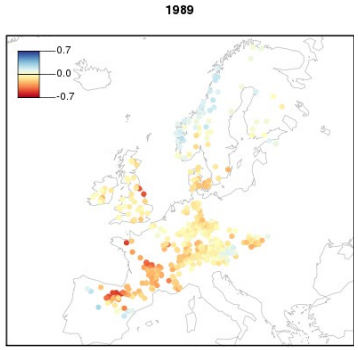

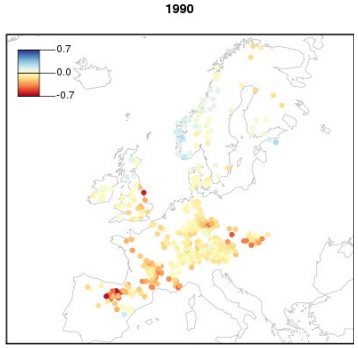

- **Onset of a draught: Drop in δ_{LVD}**
- **Dry and warm years:**
  - More homogeneous annual cycles within the domain

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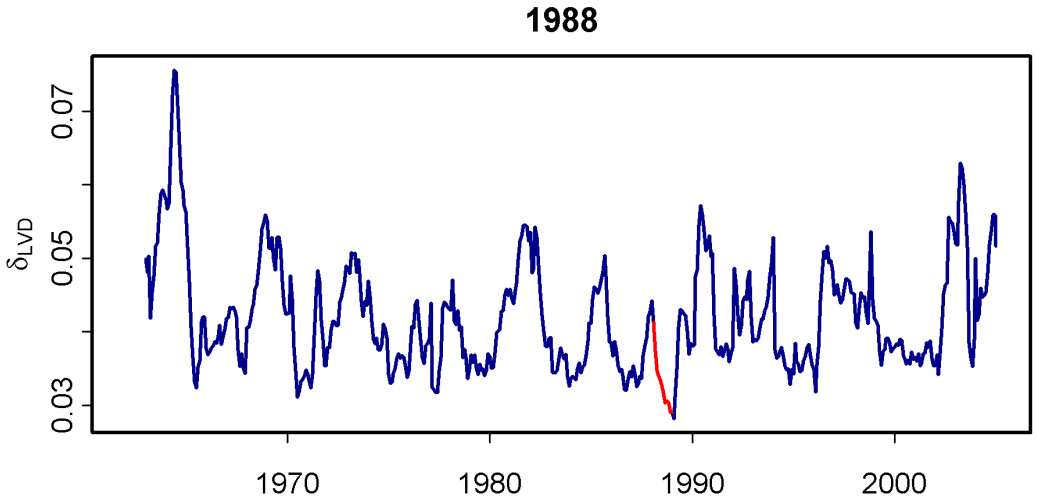

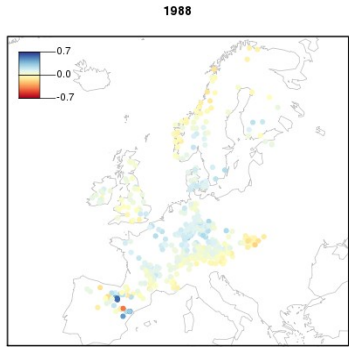

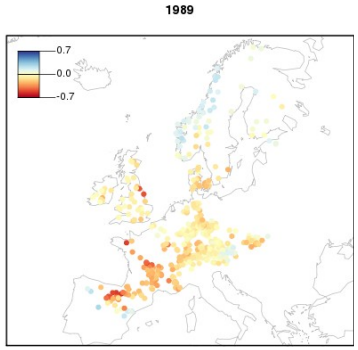

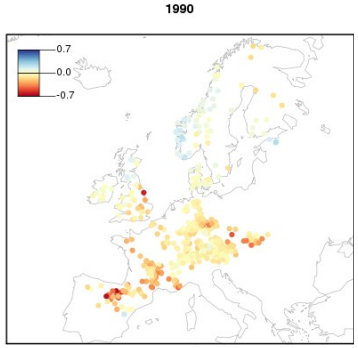

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