

Predicting reservoir inflows using water balance equations combined with particle filtering and empirical information

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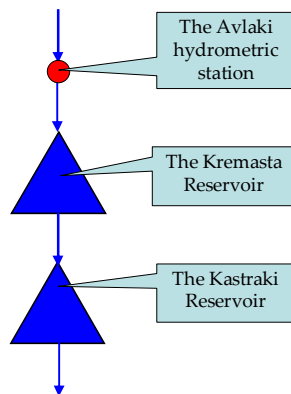
1 Motivation

1. Water balance equations are frequently used for predicting historical inflows of reservoirs; inflows are useful in many applications (e.g., the calculation of drought indices).
2. The problem:
 - All components of water balance contain uncertainty.
 - Water balance leads to incorporating uncertainty in "measured" components within the inflow estimates.
3. The question:
 - Can we jointly exploit inflows obtained through water balance and other kinds of available information so as to produce a hybrid "historical" inflow series?

2 Aim

- Investigate the effectiveness of using statistical information from multiple flow records with the purpose to produce the "historical" inflows of reservoirs (monthly series)

4 The study system



The Acheloos river basin, Western Greece

- Basin area (km²)
 - Avlaki: 1356
 - Kremasta: 3584
 - Kastraki: 4125
 - Periods of availability of monthly data (with gaps):
 - Avlaki: 1960-1994
 - Kremasta: 1950-1994
 - Kastraki: 1969-1994
- (Mamassis and Nalbantis, 1995)



3 Methodology

Initial steps

- Obtain monthly inflow observations via:
 - Water balance equations for reservoirs
 - Water level records and stage-discharge curves of hydrometric stations
- Partition each observed signal $Y(i,j,k)$ into a natural process component $X(i,j,k)$ and an error term $V(i,j,k)$, where i is year, j is month and k is the variable order in the available set

$$Y(i, j, k) = X(i, j, k) + V(i, j, k)$$

- Sample variances are partitioned into natural process variances and error variances assuming constant fraction of error variance per variable k , $p(k)$

$$\sigma_Y^2(j, k) = \sigma_X^2(j, k) + \sigma_V^2(j, k) \quad \sigma_V^2(j, k) = p(k)\sigma_Y^2(j, k)$$

$$\sigma_X^2(j, k) = (1 - p(k))\sigma_Y^2(j, k)$$

The state transition model

- The multivariate contemporaneous Periodic lag-one AutoRegressive model (PAR(1)) for monthly time series (Koutsosoyannis, 1999) is used:

$$\mathbf{X}_{i,j} = \mathbf{A}_j \mathbf{X}_{i,j-1} + \mathbf{B}_j \mathbf{Z}_{i,j}$$

- where $\mathbf{X}_{i,j}$ and $\mathbf{Z}_{i,j}$ are respectively the inflows (natural process) and the innovations as n -column vectors for n variables, j is month, i is the year and \mathbf{A}_j and \mathbf{B}_j are model parameter matrices.

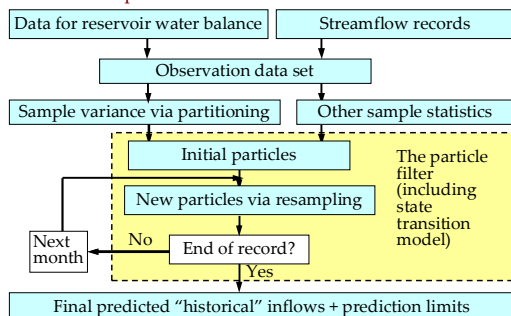
- The model preserves means, variances, third-order moments, lag-one autocovariances and lag-zero cross covariances.

The particle filter

- The Sampling Importance Resampling (SIR) filter (Gordon et al., 1993; Plaza Guingla et al., 2013) is used. Its steps are:

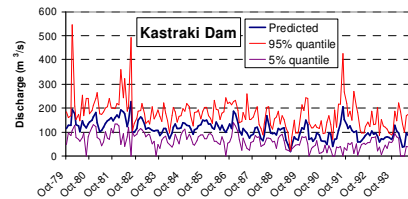
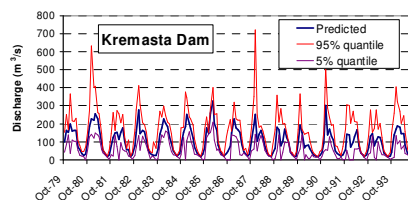
- Step 1: Draw N_s particles, $\mathbf{x}(i,j,m)$, $m = 1, 2, \dots, N_s$, from the distribution of current states conditioned by previous states (if data are missing, go to next time step)
- Step 2: Calculate particle weights by adopting a Gaussian "likelihood" function
- Step 3: Normalize weights
- Step 4: Resample particles using normalized weights
- Return to Step 1 (next time step)

Calculation steps

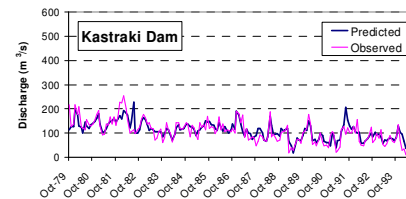
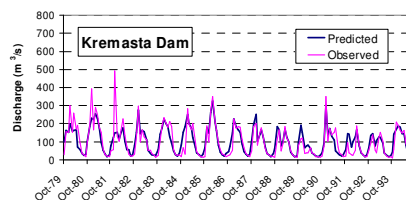


5 Results

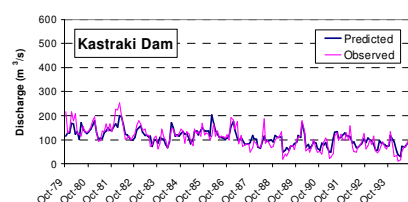
Case with low error in all variables k ($p(k) = 0.05$): Predictions and prediction limits



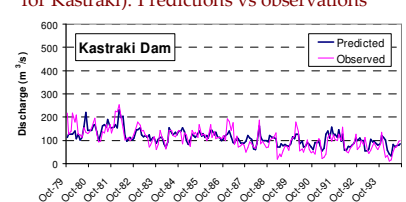
Case with low error in all variables k ($p(k) = 0.05$): Predictions vs observations



Case with high error in all variables k ($p(k) = 0.20$): Predictions vs observations



Case with realistic error ($p(1) = 0.05$ for Avlaki, $p(2) = 0.10$ for Kremasta and $p(3) = 0.20$ for Kastraki): Predictions vs observations



6 Conclusions

1. Monthly reservoir inflows are frequently assessed through using water balance equations without considering errors involved.
2. A methodology to tackle this is proposed which is easy to apply since: (1) it is based on well-known methods for multivariate stochastic modelling of time series and particle filtering; (2) it requires only the assumption on, or the empirical assessment of, the amount of error (expressed as a fraction of variance) in each flow variable involved; (3) it handles missing data.
3. In spite of making intensive use of Monte Carlo simulation the proposed computational scheme is efficient for modern multi-core computer processors.
4. The proposed methodology allows the generation of "historical" reservoir inflow samples that incorporate information also from other reservoirs or hydrometric stations thus mitigating the effect of uncertainty in outputs of reservoir water balance studies.

References

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- Koutsosoyannis, D., Optimal decomposition of covariance matrices for multivariate stochastic models in hydrology, *Water Resour. Res.*, 35 (4), 1219–1229, 1999.
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